Course Plan

1. Course Information

Course Title	Engineering Mathematics-I
Department	Mathematics
Course type	Core
Program level	B.Tech
Year and semester	I/I
Contact hours	45
Facilities required	Classroom with Black/White/Green Board, Projector and Screen, Computer/ Laptop/Tablet, Internet or Wi-Fi, Mobile Phone (optional)
Syllabus	Annexure-I

Pre requisite: Basic knowledge of differentiation, partial differentiation, integration, Geometry and Vector calculus

PEDAGOGY, TECHNOLOGY & ASSESSMENT

The learning outcome focuses on analytical level of thinking. MOODLE (or similar learning management systems) can be an effective technology, which can be used in classrooms, especially, large classrooms for facilitating learning. Teachers are encouraged to get training on how to use MOODLE online and inter or intra net should be available in classrooms.

CO 1: Analyse the existence of limits, continuity, partial derivatives at a point using mathematical definition and **apply** those concepts to evaluate them.

Model Instructional strategy to accomplish the CO 1:

LO	**Pedagogical Decision**	**Brief Description**	**Sample Technology**
Find **Justify** **Evaluate**	Individual problem solving by **Scaffolding**	• Reading materials are uploaded in the Moodle beforehand. • Problems are shown on PPT, students are asked to solve them. They find some functions have limit, some	PPT with Pictures, videos, simulation and animation *tailored* for the topic during discussion.

		not etc. They are motivated to enquire why that happens. • Discusses using videos, simulations or animations (pausing at important concepts, changing parameters and so on)	

Suggested websites and videos CO1

For limit, continuity and partial derivatives

https://www.youtube.com/watch?v=6bkFDru-g-M (6'18")

https://www.youtube.com/watch?v=ZjgHOreTUmw (5'16")

https://www.youtube.com/watch?v=d0SPcQNy78E&t=315s (11'17")

https://www.youtube.com/watch?v=ftAuCXNAvtE&t=301s (6'42")

http://mathonline.wikidot.com/limits-of-functions-of-two-variables

https://www.youtube.com/watch?v=GkB4vW16QHI (5'23'')

http://mathinsight.org/partial_derivative_introduction Nykamp DQ

Nykamp DQ, "Introduction to partial derivatives." From *Math Insight*. http://mathinsight.org/partial_derivative_introduction

https://www.youtube.com/watch?v=kCr13iTRN7E (8'43")

ASSESSMENT PLAN CO 1:

Type of assessment	**Frequency of assessment**	**Delivery from the learner**	**Data collection**	**Learning Verification**	**Decision making**
Formative	Each class (Number of classes = 9)	a) Solve problems individually	a) Online through Moodle	a) (i) Learners see the correct answer in the feedback given; (ii) Class performance will be analyzed by Moodle	
		b) Homework problems	b) Hard copy submission	b) Evaluates the hard copy with comments	

				and returns to students	
	At the end of 9 classes	c) Solve one assignment	c) Hard copy submission	c) Evaluates the hard copy; determines class average; gives back the assignment sheets to the class and discusses the mistakes	

Sample examination questions: CO 1:

Course Outcome 1	
Analyse the existence of limits, continuity, partial derivatives at a point using mathematical definition and **apply** those concepts to evaluate them.	
Sl.No.	Questions
1	Show the existence or otherwise of the following limit: $\lim_{P\to P_0} \frac{2y}{x^2+y^2+2y}$ where $P(x,y)$ and $P_0=(0,0)$ and also evaluate the limit if it exists.
2	Show the existence or otherwise of the following limit: $\lim_{P\to P_0} \frac{x^4y-3x^2y^3+y^5}{(x^2+y^2)^2}$ where $P(x,y)$ and $P_0=(0,0)$ and also evaluate the limit if it exists.
3	Determine if $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ exists and justify.
4	If $f:R^2\to R$ is a function continuous at (0,0) with $f(0,0)=1$, give a value of ε in the $\varepsilon-\delta$ definition of continuity so that $f>\frac{99}{100}$ in a neighborhood of (0,0). Justify your answer briefly.
5	Let $f(x,y)=\begin{cases}\frac{\sin(x+y-2)}{x-y} & if\ \ x\neq y\\ 1 & if\ \ x=y\end{cases}$. Is f continuous at (1,1)? Justify.
6	If $f:D\to R$ is a function such that f_x and f_y are continuous on D, then prove

	that f is continuous on D.
7	Let $f(x,y) = In\left(\frac{3x^2 + x^2y^2 + 3y^2}{x^2 + y^2}\right)$ for $(x,y) \neq (0,0)$. Define $f(0,0)$ in a way that extends f to be continuous at (0,0).
8	Let $f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4} & if \ (x,y) \neq (0,0) \\ 0 & if \ (x,y) = (0,0) \end{cases}$ Is f differentiable at (0, 0)? Justify.
9	Let $f(x,y) = \frac{3x^2y - y^3}{x^2 + y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Compute $f_y(x,0)$ and $f_y(0,0)$. Determine whether f_y is continuous at (0,0).
10	Do the first partial derivatives exist at (0,0) for the function $f(x,y) = \frac{x^3 + y}{x^2 + y^2}$ if $(x,y) \neq (0,0)$ with $f(0,0) = 0$?
11	Let $f : R^2 \to R$ be such that $f(\alpha(u,v) + \beta(x,y)) = \alpha f(u,v) + \beta f(x,y)$ for all $\alpha, \beta \in R$ and $(u,v),\ (x,y) \in R^2$ show that $\frac{\partial f}{\partial x}(1,2) = f(1,0)$
12	Given that $f(x,y) = \frac{x\lvert y\rvert + y\lvert x\rvert}{x^2 + y^2}$, $(x,y) \neq (0,0)$, does the mixed second partial derivative $f_{yx} = \frac{\partial^2 f}{\partial x \partial y}$ exist at (1, -1)? Justify your answer briefly and give the value of $f_{yx}(1,-1)$ (if it exists).
13	Suppose $u(x,y)$ and $v(x,y)$ satisfy $u_x = v_y$ and $u_y = -v_x$ for all (x,y). Prove that $v_\theta = ru_r$ where $x = r\cos\theta$ and $y = r\sin\theta$.
14	Find all the second order partial derivative of $g(x,y) = x^2y + \cos(y) + y\sin(x)$.

15	Suppose that $f(x,y,z,w)=0$ and $g(x,y,z,w)$, where z and w are differentiable functions of independent variables x and y . If $f_z g_w - f_w g_z \neq 0$, then show the following: $$\frac{\partial z}{\partial x}=\left(\frac{f_x g_w - f_w g_x}{f_z g_w - f_w g_z}\right)$$

CO2: Analyse the concepts of partial derivatives and **apply** them to obtain

1. directional derivatives and gradient,
2. the equations of tangent planes and normal lines,
3. Taylor series expansions and
4. extreme values of a function.

Model Instructional strategy to accomplish the CO 2:

LO	Pedagogical Decision	Brief Description	Sample Technology
Find **Compute** **Show**	Problem solving by **ACL**	• Reading materials, narrated PPT and some relevant videos are uploaded in the Moodle beforehand. • Problems are shown on PPT • Students are grouped. Each group works out work out scenario problems similar to the following. **Example Problem:** Suppose that the Celsius temperature at a point (x,y,z) on the sphere $x^2+y^2+z^2=1$ is $T=400xyz^2$. Locate the highest and least temperatures on the sphere. • Problems are shown on PPT, students are asked to solve them by group work. Exchanging the complete working out of the problems of one group with another group and then mutually check the	Teacher can use PPT with pictures final feedback

		answers. • One student at random from each group can be asked to work out the problem on the board. Teacher gives feedback.	

Suggested websites and videos CO2

https://www.youtube.com/watch?v=TEB2z7ZlRAw (10’3’’)

https://www.youtube.com/watch?v=UX1uENJvuO0 (9’17”)

https://www.youtube.com/watch?v=KVwgFvmHnBM (9’50”)

https://www.geogebra.org/m/C7sq5Vbh

ASSESSMENT PLAN CO 2:

Type of assessment	Frequency of assessment	Delivery from the learner	Data collection	Learning Verification	Decision making
Formative	Each class (Number of classes = 9)	a) Evaluation of the problems of the group of students assigned to them, using a scheme of evaluation	a) Hard copy of the peer evaluation submission	a) Feedback by the teacher based on her evaluation and peer evaluation	
		b) Homework problems	b) Hard copy submission	b) Evaluates the hard copy with comments and returns to students	
	At the end of 9 classes	c) Solve one assignment	c) Hard copy submission	c) Evaluates the hard copy; determines class average; gives back the assignment sheets	

				to the class and discusses the mistakes	

Sample examination questions: CO 2:

Course Outcome 2	
Analyse the concepts of partial derivatives and **apply** them to obtain 1. directional derivatives and gradient, 2. the equations of tangent planes and normal lines, 3. Taylor series expansions and 4. extreme values of a function.	
Sl.No.	Questions
1	Compute the directional derivatives (when they exist) at (1,1) for the function $f(x,y)=\frac{x+y}{x-y}$ if $x\neq y$ and $f(x,y)=x+y$ if $x=y$, along the directions $\vec{u}=(\vec{i}+\vec{j})/\sqrt{2}$ and $\vec{v}=(\vec{i}-\vec{j})/\sqrt{2}$ where $\vec{i}$ and $\vec{j}$ denote the unit vectors along the positive x and y directions respectively
2	Is there a direction in which the directional derivative of $f(x,y)=x^2-3xy+y^2$ at (1,2) equals 14? Justify.
3	Find the directional derivative of f at (0, 0) in the direction of $\frac{1}{\sqrt{2}}\vec{i}+\frac{1}{\sqrt{2}}\vec{j}$.
4	Let $g(x,y)=\frac{\lVert x\rvert-\lvert y\rVert-\lvert x\rvert-\lvert y\rvert}{\sqrt{x^2+y^2}}$ for $(x,y)\neq(0,0)$ and $g(0,0)=0$. Determine whether the directional derivative of $g(x,y)$ in the direction of $\frac{1}{\sqrt{2}}\vec{i}+\frac{1}{\sqrt{2}}\vec{j}$ exists at (0,0).
5	Find the equation of the tangent plane to the surface $z=G(x,y)$ at any arbitrary point on the y-axis, where $z=G(x,y)$ is given implicitly by the equation: $e^x yz^2+2yz+\sin x=0$.

6	Find the equation of the tangent plane and normal line at (1,1,1) to $x^2+y^2+z^2+xy+xz=5$
7	In the Taylor expansion at the origin given by $\frac{1}{1-x-y}=\sum_n\sum_m C_{(n,m)}x^n y^m$ what is the value of $C_{(n,m)}$?
8	Find the local extrema of $f(x,y)=x^2-2x+2y^2+4y-2$
9	Find all the local maxima, local minima and saddle points for the function $f(x,y)=2x^3+2y^3-9x^2+3y^2-12y$
10	Classify the critical (stationary) points of $f(x,y)=2x^3+6xy^2-3y^3-150x.$
11	Find the critical points of $F(x,y)=x^3+y^3-3axy$ for $a\neq 0$ and classify them.
12	Find the critical points of $f(x,y)=x+y+(1/(xy))$ in the region given by $\{(x,y)\in R^2 : x>0,\ y>0\}$ and identify their nature
13	Find the least distance from (0,0) to $x^2+8xy+7y^2=225$
14	Using the method of Lagrange Multipliers, find the points on the surface z^2-xy closest to the origin.

CO 3: Analyse the concepts of double and triple integrals of continuous functions on a finite domain and **apply** them to find the area and volume.

Model Instructional strategy to accomplish the CO 3:

LO	Pedagogical Decision	Brief Description	Sample Technology
Find **Evaluate** **Compute**	Problem solving by **Think-Pair-Share**	• Reading materials and some relevant videos are uploaded in the Moodle beforehand. • Problems are shown on PPT in the class • The preliminary essentials required for working out each problem is done by **Think-Pair and Share.** • One or two students randomly chosen from each group explain and work out the solution of the problem	Students can use PPT with Pictures, simulation or animation to explain their presentation

		on the board while other groups and the teacher watch and give feedback.	

Suggested websites and videos CO3

https://in.video.search.yahoo.com/search/video?fr=mcafee&p=double+integration+animation#id=1&vid=16b8bdbdb936cfa5bc7bc80e717fa658&action=click (16')

https://www.youtube.com/watch?v=i4L5XoUBD_Q (1'30'')

https://www.youtube.com/watch?v=rbqWHbxmVUI (8'22'')

https://www.youtube.com/watch?v=8gbphumzbSI (7'06'')

http://mathinsight.org/triple_integral_examples

Nykamp DQ, "Triple integral examples." From *Math Insight*. http://mathinsight.org/triple_integral_examples

http://www.wolframalpha.com/widgets/view.jsp?id=4962568f78ad74628f0096d1d445b525 (triple integral calculator)

https://www.youtube.com/watch?v=7hX1DHq7sGw (4'41'')

http://mathinsight.org/double_integral_change_variable_illustrated_example

Nykamp DQ, "Illustrated example of changing variables in double integrals." From *Math Insight*. http://mathinsight.org/double_integral_change_variable_illustrated_example

https://in.images.search.yahoo.com/search/images;_ylt=A2oKmJsMa8ZYug0AHxi7HAx.;_ylu=X3oDMTByYmJwODBkBGNvbG8Dc2czBHBvcwMxBHZ0aWQDBHNlYwNzYw--?p=Cylindrical+Coordinates&fr=mcafee

http://mathinsight.org/cylindrical_coordinates

Nykamp DQ, "Cylindrical coordinates." From *Math Insight*. http://mathinsight.org/cylindrical_coordinates

https://in.images.search.yahoo.com/search/images;_ylt=A2oKmL71lcdYcnoArbK7HAx.;_ylu=X3oDMTByYmJwODBkBGNvbG8Dc2czBHBvcwMxBHZ0aWQDBHNlYwNzYw--?p=Spherical+Coordinates&fr=mcafee

http://mathinsight.org/spherical_coordinates

Nykamp DQ, "Spherical coordinates." From *Math Insight*. http://mathinsight.org/spherical_coordinates

ASSESSMENT PLAN CO 3:

Type of assessment	Frequency of assessment	Delivery from the learner	Data collection	Learning Verification	Decision making
Formative	Each class (Number of classes = 9)	a) Evaluation of the problems of the group of students by other groups, using a scheme of evaluation	a) Hard copy of the peer evaluation submission	a) Feedback by the teacher based on her evaluation and peer evaluation	
		b) Homework problems	b) Hard copy submission	b) Evaluates the hard copy with comments and returns to students	
	At the end of 9 classes	c) Solve one assignment	c) Hard copy submission	c) Evaluates the hard copy; determines class average; gives back the assignment sheets to the class and discusses the mistakes	

Sample examination questions: CO 3:

Course Outcome 3	
Analyse the concepts of double and triple integrals of continuous functions on a finite domain and **apply** them to find the area, volume.	
Sl.No.	Questions
1	If R is the square region in the plane bounded by the lines $\lvert x\rvert + \lvert y\rvert = 1$, evaluate $\iint_R y\,dx\,dy$

2	Evaluate $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dydx$.
3	Change the order of integration and evaluate $\int_0^2 \left(\int_{x^2}^4 dy \right) dx$
4	Sketch the domain and change the order of the integration $\int_{-1}^2 \int_{-x}^{2-x^2} f(x,y) dydx$.
5	Transform the following integral into polar coordinates: $\int_0^a \left(\int_0^x f(x,y) dy \right) dx$
6	Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y}(y-2x)^2 dydx$ using change of variables.
7	If the curve with equation $f(x,y) = y^2(1-x^2) - x^4 = 0$ is expressed in the polar form $r = F(\theta)$, then write the expression for $F(\theta)$.
8	Let A be the region in the first quadrant of the xy-plane bounded by the hyperbolas $xy=1;\ xy=9$ and the lines $y=x;\ y=4x$. Using the transformation $x=u/v;\ y=uv$ with u, v both positive , evaluate the integral. $\iint_A \left(\sqrt{\frac{y}{x}} + \sqrt{xy} \right) dxdy$
9	If $A = \{(x,y) \in R^2 : 3x^2 + 2y^2 \le 6\}$ then evaluate $\iint_A \lvert xy \rvert dxdy$
10	Let D be the region enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ prove that $\pi(e^4 - 1) \le \iint_D e^{(x^2+y^2)} dA \le \pi(e^9 - 1)$.
11	Find the area of the loop of the polar curve $x = 2(1-\sin\theta)\sqrt{\cos\theta}$ in the polar region given by $-\pi/2 \le \theta \le \pi/2$.
12	Find the area of the region that lies outside the circle $r=1$ and inside $r = 2\sin\theta$.
13	Evaluate $\iiint_R (x^2+y^2+z^2)^{3/2} dV$ where R is the solid bounded by sphere $x^2+y^2+z^2=1$, the cones $z=\sqrt{x^2+y^2}$ and $z=\sqrt{3x^2+3y^2}$

14	Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\frac{x^2+y^2}{2}} \frac{z}{\sqrt{x^2+y^2}} dzdydx$
15	Compute the volume of the solid bounded by $x^2+y^2+z^2=4$ and $x^2+y^2=2y$.
16	Using a suitable triple integral, find the volume of the solid region bounded from below by the cone $z=\sqrt{x^2+y^2}$ and from above by the sphere $x^2+y^2+z^2=4$.
17	If V is the solid region in 3-space bounded by $x=0=y,\ z=0$ and $15x+10y+6z=30$ then find $\iiint_V dxdydz$.
18	Find the volume of the region bounded below by the sphere $\rho = 2\cos\phi$ and above by the cone $\phi = \pi/4$.

CO 4: Analyse the concepts of line and surface integrals of continuous vector point functions and **apply** them to find the circulation, work done, conservative field, surface area etc.

Model Instructional strategy to accomplish the CO 4:

LO	Pedagogical Decision	Brief Description	Sample Technology
Find **Evaluate** **Compute**	Problem solving either individual or in group **(Flipped class room)**	• The topic and related materials are given to students in advance through Moodle, students are asked to study before coming to the next class. • Then students are asked to solve problems either in groups or individually. • Teacher checks the problem randomly.	Videos, soft copy of reading material are provided to the students.

Suggested websites and videos CO 4

http://mathinsight.org/parametrized_curve_introduction

Nykamp DQ, "An introduction to parametrized curves." From *Math Insight*. http://mathinsight.org/parametrized_curve_introduction

http://mathinsight.org/parametrized_curve_introduction (2minutes)

http://mathinsight.org/conservative_vector_field_introduction (2 minutes)

Nykamp DQ, "An introduction to conservative vector fields." From *Math Insight*. http://mathinsight.org/conservative_vector_field_introduction

https://www.youtube.com/watch?v=4XLq-BWK5NY (9'46'')

http://www.shodor.org/interactivate/activities/FunctionRevolution/

http://mathinsight.org/polar_coordinates

Nykamp DQ, "Polar coordinates." From *Math Insight*. http://mathinsight.org/polar_coordinates

http://mathinsight.org/cylindrical_coordinates

Nykamp DQ, "Cylindrical coordinates." From *Math Insight*. http://mathinsight.org/cylindrical_coordinates

http://mathinsight.org/spherical_coordinates

Nykamp DQ, "Spherical coordinates." From *Math Insight*. http://mathinsight.org/spherical_coordinates

http://tutorial.math.lamar.edu/Classes/CalcIII/SurfIntVectorField.aspx

http://tutorial.math.lamar.edu/Classes/CalcIII/SurfaceIntegrals.aspx

ASSESSMENT PLAN CO 4:

Type of assessment	**Frequency of assessment**	**Delivery from the learner**	**Data collection**	**Learning Verification**	**Decision making**
Formative	Each class (Number of classes = 9)	a) Answers from all the students	a) Hard copy submission	a) Evaluates the hard copy with comments and returns to students	
		b) Homework problems	b) Hard copy submission	b) Evaluates the hard copy with comments and	

				returns to students	
	At the end of 9 classes	c) Solve one assignment	c) Hard copy submission	c) Evaluates the hard copy; determines class average; gives back the assignment sheets to the class and discusses the mistakes	

Sample examination questions: CO 4:

Course Outcome 4	
Analyse the concepts of line and surface integrals of continuous vector point functions and **apply** them to find the circulation, work done, conservative field, surface area etc.	
Sl.No.	Questions
1	Let $\vec{F}(x,y)=(3+2xy)\vec{i}+(x^2-3y^2)\vec{j}$. Does there exist a function f such that 2 $\nabla f=\vec{F}$? Hence or otherwise, evaluate $\int_C \vec{F}.d\vec{r}$, where C is the curve given by $\vec{r}(t)=e^t\sin t\,\vec{i}+\cos t\,\vec{j}, t$ varying from 0 to π.
2	Evaluate $\int_C \vec{F}.d\vec{r}$, where $\vec{F}=z\vec{i}+xy\vec{j}-y^2\vec{k}$ along the curve $C:\vec{r}(t)=t^2\vec{i}+t\vec{j}+t\vec{k}$: $0\le t\le 1$
3	Show that $\vec{F}=(e^x\cos y+yz)\vec{i}+(xz-e^x\sin y)\vec{j}+(xy+z)\vec{k}$ is conservative and find a potential function for it.
4	Find the flow of the velocity field $\vec{F}=(x+y)\vec{i}-(x^2+y^2)\vec{j}$ along the upper half of the circle $x^2+y^2=1$ from $(1,0)$ to $(-1,0)$ in the xy plane
5	Find the circulation of $\vec{F}=2x\vec{i}+2z\vec{j}+2y\vec{k}$ around the closed path $\overrightarrow{r(t)}=\cos t\vec{i}+\sin t\vec{j}+t\vec{k}, 0\le t\le\frac{\pi}{2}$.
6	Find the work done by the force $\vec{F}=(3x^2-3x)\vec{i}+3z\vec{j}+\vec{k}$ along the curved path $\overrightarrow{r(t)}=t\vec{i}+t^2\vec{j}+t^4\vec{k}, 0\le t\le 1$

7	Compute the area of that part of the cylinder $x^2+y^2=1$ which lies between the planes $z=2x$ and $z=0$.
8	Find the area of the upper portion of the cylinder $x^2+z^2=1$ that lies between the planes $x=\pm\frac{1}{2}$ and $y=\pm\frac{1}{2}$.
9	Evaluate $\iint_\sigma \vec{F}.\hat{n}\,d\sigma$, where $\vec{F}(x,y,z)=x\vec{i}+y\vec{j}-z\vec{k}$ and σ is the surface of the cone $z=\sqrt{x^2+y^2}$ between the planes $z=0$ and $z=1$.
10	Evaluate $\iint_S xz d\sigma$, where S is the portion of the sphere centered at the origin and of radius 3 with $x\le 0,\ y\ge 0$ and $z\ge 0$.
11	Integrate $g(x,y,z)=xyz$ over the surface of the rectangular solid bounded by the planes $x=\pm a, y=\pm b, z=\pm c.$

CO 5: Analyse Green's theorem, Stoke's theorem and Gauss divergence theorem and **apply** them in various given situations.

Model Instructional strategy to accomplish the CO 5:

LO	Pedagogical Decision	Brief Description	Sample Technology
Find **Verify** **Apply**	Problem solving by **ACL**	• Topics are given to the students beforehand, students should search for the resources and find out the materials • Seminar presentation of the topic by a group. Each group presents the topic allotted to them while other groups and the teacher observe and ask questions. Teacher gives feedback.	Student uses PPT with pictures or animations for presentation

Suggested websites and videos CO 5

https://www.youtube.com/watch?v=gGXnILbrhsM (10’30’’)

https://www.youtube.com/watch?v=sSyPAAyL8nQ (7’6’’)

https://www.youtube.com/watch?v=YXf3aKxgELY (11’13”)

https://www.youtube.com/watch?v=vXsZmEAZ2SI (9’36”)

https://www.youtube.com/watch?v=_ubpnd8F85g (6’13)

https://www.youtube.com/watch?v=WBc-hmE_TCo(10’29”)
https://www.youtube.com/watch?v=asyIsn59Lnc (10’40’’)

ASSESSMENT PLAN CO 5:

Type of assessment	**Frequency of assessment**	**Delivery from the learner**	**Data collection**	**Learning Verification**	**Decision making**
Formative	Each class (Number of classes = 9)	a) Evaluation of the presentation of the group of students by other groups, using a scheme of evaluation	a) Hard copy of the peer evaluation submission	a) Feedback by the teacher based on her evaluation and peer evaluation	
		b) Homework problems	b) Hard copy submission	b) Evaluates the hard copy with comments and returns to students	
	At the end of 9 classes	c) Solve one assignment	c) Hard copy submission	c) Evaluates the hard copy; determines class average; gives back the assignment sheets to the class and discusses the mistakes	

Sample examination questions: CO 5:

Course Outcome 5	
Analyse Green's theorem, Stoke's theorem and Gauss divergence theorem and **apply** them in various given situations.	
Sl.No.	Questions
1	Verify Green's theorem given that $\vec{F}(x,y) = y^2\vec{i} + 3xy\vec{j}$ and C is the boundary of the semi-annular region R in the upper half-plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.
2	Evaluate the $\oint_C (3ydx + 2xdy)$; C : the boundary of $0 \le x \le \pi, 0 \le y \le \sin x$ by applying Green's theorem:.
3	Evaluate the line integral $\oint_C y^2dx + x^2dy$ using Green's Theorem, where C is the triangle with vertices (0,0), (1,1) and (1,0) oriented in the counter clockwise direction.
4	Use Green's theorem area formula to find the areas of the region enclosed by C which is given by : $\vec{f}(t) = (\cos^3 t)\vec{i} + (\sin^3 t)\vec{j},\ 0 \le t \le 2\pi$
5	Using Stokes' theorem, evaluate the line integral $\oint_C 2ydx + zdy + 3ydz$ where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = 6z$ and the plane $z = x + 3$ oriented in the clockwise sense, as seen from the origin.
6	Use Stokes' Theorem to evaluate: $\iint_S curl\,\vec{F}.\hat{n}\,d\sigma$ where S is the surface $z = x^2 + y^2;\ 0 \le z \le 4,\ \vec{F} = 3z\vec{i} + 4x\vec{j} + 2y\vec{k}$ and $\hat{n}$ is the unit outward normal.
7	Use Stoke's theorem to calculate $\oint_C \vec{F}.d\vec{r}$ in the counterclockwise direction when viewed from above $\vec{F} = 2y\vec{i} + 3x\vec{j} - z^2\vec{k};\ C: x^2 + y^2 = 9$ in the xy-plane.
8	Use Stoke's Theorem to calculate the flux of the curl of F across the surface S in the direction of the outward unit normal n. $\vec{F} = (y-z)\vec{i} + (z-x)\vec{j} + (x+z)\vec{k}$ $S: \vec{r}(r,\theta) = (r\cos\theta)\vec{i} + (r\sin\theta)\vec{j} + (9-r^2)\vec{k};\ 0 \le r \le 3, 0 \le \theta \le 2\pi$

9	Using the divergence theorem, evaluate the surface integral $\iint_{\sigma} F.\hat{n}\, d\sigma$ where $\vec{F}(x,y,z) = 2xz\hat{i} - xy\hat{j} - z^2\hat{k}$ and σ is the surface of the wedge cut from the first octant by the intersection of the surfaces $x^2 + y^2 = 1$ and $y + z = 1$
10	Verify Gauss Divergence Theorem for $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ and the region R in R^3 bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$.
11	Use Divergence theorem to find the outward flux of F across the boundary of D: $\vec{F} = x^2\vec{i} - 2xy\vec{j} + 3xz\vec{k}$; D: the region cut from the first octant by the sphere $x^2 + y^2 + z^2 = 4$.
12	Show that the flux of the position vector field $x\vec{i} + y\vec{j} + z\vec{k}$ outward through a smooth closed surface S is three times the volume of the region enclosed by the surface.

QUIZZES and Summative assessment

Formative evaluation	End of first 13 classes	TEST PAPER I	Hard copy	Evaluates the hard copy, grades them, returns the paper and discusses Weightage for this portion: 20 %
Formative evaluation	End of next 13 classes	TEST PAPER I	Hard copy	Evaluates the hard copy, grades them, returns the paper and discusses Weightage for this portion: 20 %

Summative assessment	At the end of the course	End semester Examination	Hard copy	Evaluates the hard copy and grades. Weightage for this portion: 60 %

Annexure- I

Syllabus: First Semester/ Engineering Mathematics/All Branches

Unit-1	Regions in plane, level curves and level surfaces, limit, continuity, partial derivatives,
Unit-II	Directional derivatives, gradient, tangent planes and normal lines, Taylor series, extreme values, Lagrange multipliers.
Unit-III	Double integrals in Cartesian and polar coordinates, triple integral, change of variables, multiple integrals in cylindrical and spherical coordinates, area enclosed by plane curves, volume of solids.
Unit-IV	Gradient, divergence, curl, line integral, conservative fields, surface area, surface integral.
Unit-V	Green's theorem in plane, Stokes' theorem, Gauss Divergence theorem.

Text Book: G.B. Thomas Jr., M.D. Weir and J.R. Hass, Thomas Calculus, Pearson Education, 2009.

Reference Books:

1. **E. Kreyszig, Advanced Engineering Mathematics (Tenth Edition), John Wiley & Sons.**

2. **N.Piskunov, Differential and integral Calculus Vol 1, 1-2, Mir publishers,**

   Moscow jointly with CBS Publishers&Distributers, India

Mapping of content and Learning Outcomes

Units	Content	Course Outcomes
Unit-1	Regions in plane, level curves and level surfaces, limit, continuity, partial derivatives.	**Analyse** the existence of limits, continuity, partial derivatives at a point using mathematical definition and **apply** those concepts to evaluate them.
Unit-II	Directional derivatives, gradient, tangent planes and normal lines, Taylor series, extreme values, Lagrange multipliers	**Analyse** the concepts of partial derivatives and **apply** them to obtain 1. directional derivatives and gradient, 2. the equations of tangent planes and normal lines, 3. Taylor series expansions and 4. extreme values of a function.
Unit-III	Double integrals in Cartesian and polar coordinates, triple integral, change of variables, multiple integrals in cylindrical and spherical coordinates, area enclosed by plane curves, volume of solids.	**Analyse** the concepts of double and triple integrals of continuous functions on a finite domain and **apply** them to find the area and volume.
Unit-IV	Gradient, divergence, curl, line integral, conservative fields, surface area, surface integral.	**Analyse** the concepts of line and surface integrals of continuous vector point functions and **apply** them to find the circulation, work done, conservative field, surface area etc.
Unit-V	Green's theorem in plane, Stokes' theorem, Gauss Divergence theorem.	**Analyse** Green's theorem, Stoke's theorem and Gauss divergence theorem and **apply** them in various given situations.

- **Most of the questions in the document are taken from the tutorial and assignment sheets, quiz and end semester question papers of the First semester paper "Functions of Several Variables" taught at IIT Madras, over the years and from the text book G.B. Thomas Jr., M.D. Weir and J.R. Hass, Thomas Calculus, Pearson Education, 2009.**
- **We acknowledge the open source contents which are used as reference materials in this document.**